

## LINEARIZATION TECHNIQUE FOR SOLVING QUADRATIC SET PACKING AND PARTITIONING PROBLEMS

RASHMI GUPTA<sup>1</sup> & RATNESH RAJAN SAXENA<sup>2</sup>

<sup>1</sup>Research Scholar, Department of Mathematics, University of Delhi, Delhi, India

<sup>2</sup>Associate Professor, Department of Mathematics, Deen Dayal Upadhyaya College, University of Delhi, Delhi, India

### ABSTRACT

Set Packing and Set Partitioning Problems belong to the class of 0-1 integer programming problems that are NP-complete. Packing problems are a class of optimization problems in mathematics which involves attempting to pack objects, as densely as possible but optimally, many applications arise having the packing and partitioning structure. Delivery and routing problems, scheduling problems and location problems, switching theory, wireless network design, VLSI circuits and line balancing often take on a set packing structure, if one wishes to satisfy as much demand as possible without creating conflict and, if every customer must be served by exactly one server, the problem takes on a set partitioning format. The Set Packing Problem has a dual covering problem, which asks how many of the same objects are required to completely cover every region of the container, where the objects are allowed to overlap. In this paper a linearization technique is developed to solve packing problems with non-linear objective function, in particular the quadratic objective function. This is an extension of the work done by Saxena and Arora [19]. The algorithms redeveloped in this paper are supported by numerical examples.

**MATHEMATICAL SUBJECT CLASSIFICATION:** 65K05, 90C10

**KEYWORDS:** Set Packing, Quadratic Set Packing, Set Partitioning

### 1. INTRODUCTION

Packing problems can be related to real life packaging, storage and transportation issues. The set packing and partitioning problems with linear functions has been studied by Garfinkel and Nemhauser ([9], [14]), Roth[17], Lemke, Salkin and Spielberg [13], Bellmore and Ratliff [2], Chvatal [6] and Balas ([2], [3]). They discussed and developed various enumeration techniques for these problems.

In the year 1977, Arora and Puri [1] discussed the enumeration technique for the set covering problem with linear fractional function as its objective functions. Extending their work, in the year 1998, Saxena and Arora[20] discussed a cutting plane technique for multi-objective fractional Set Covering Problem, Saxena and Arora ([18], [19]) also discussed linearization technique for solving the Quadratic Set Covering Problem and an enumeration technique for solving multi objective linear Set Covering Problem. Later in the year 2013 Saxena and Gupta ([10], [22]) worked on Set Covering, Packing and Partitioning problems with Quadratic Fractional and Linear Fractional objective functions

There are many applications of quadratic set packing problems in real life.

One of which is coming from medicine, is the following: we want to determine if an illness can be related to some other medical parameters of patients, such as finding a correlation between heart attack and cholesterol for instance.

To analyze this, we collect data on these parameters for both ill and healthy people. More formally, each person gives data on several criteria (weight, cholesterol, etc.) and is represented as a point in  $R^n$  ( $n$  is the number of criteria). Thus we have a set  $S^+$  of positive points (ill people) and a set  $S^-$  of negative points (healthy people). A first step in the analysis of the data produces a collection of the positive and negative patterns. A positive (respectively negative) pattern is a hypercube in  $R^n$  which contains no negative points (no positive points). This collection is such that every point is packed. From a medical point of view, we would like to find a sub-collection of patterns such that, every point is packed and the volume of interactions between positive and negative patterns is as large as possible (for criteria relevance).

Now define  $h_{ij}$  as the volume of the interaction between the positive pattern  $S_i^+ \in S^+$  and the negative pattern  $S_j^- \in S^-$ , then the problem is to find a packaging of all points such that  $\sum_{i,j} h_{ij} S_i^+ S_j^-$  is to be maximized.

The above instance results in a Quadratic Set Packing Problem.

In this paper the objective function would be a quadratic function. The present technique involves the linearization of a Quadratic Set Packing and Partitioning problems till the optimal solution is obtained.

Consider the class of problems having the following structure:

$$\text{Min } DX$$

$$AX \geq e^T$$

$$x_j = 0 \text{ or } 1 \text{ for } j=1, \dots, n$$

Where  $A$  is a  $m \times n$  matrix of zeroes and ones,  $e^T = (1, \dots, 1)$  is a vector of  $m$  ones and  $D$  is a vector of  $n$  (arbitrary) rational components. This pure 0-1 linear programming problem is called the **Set Partitioning Problem**, when the inequalities are replaced by equations. When all of the  $\geq$  constraints are replaced by  $\leq$  constraints with maximized objective function, the problem is called the **Set Packing Problem**.

## 2. THEORETICAL DEVELOPMENT

Considering the basic structural format of the problem this section is divided into two sections:

### 2.1 Set Packing Problems

Consider a set  $I = \{1, 2, \dots, m\}$  and a set  $P = \{P_1, P_2, \dots, P_n\}$ , where  $P_j \subseteq I$  for each  $j \in J = \{1, 2, \dots, n\}$ . A subset  $J^*$  of  $J$  is said to be a pack of  $I$  if  $\bigcup_{j \in J^*} P_j = I$ , and  $j \& k \in J^*$ ,  $j \neq k$  and  $P_j \cap P_k = \emptyset$ . Let a weight  $c_j > 0$  be associated with every  $j \in J$ . The total weight of the pack  $J^*$  is equal to  $\sum_{j \in J^*} c_j$ .

The Quadratic Set Packing Problem (**QPkP**) is to find a packing of maximum weight subject to the condition that at the most one of the utility is satisfied. Mathematically, the problem is

$$(\mathbf{QPkP}) \text{ Max } Z (= f(X)) = \sum_{j=1}^n d_j x_j + \sum_{j=1}^n \sum_{k=1}^n h_{jk} x_j x_k$$

$$\text{Subject to } \sum_{j=1}^n a_{ij} x_j \leq 1, i \in I \quad (2.1)$$

$$x_j = 0 \text{ or } 1, j \in J \quad (2.2)$$

$$\text{Where } x_j = \begin{cases} 1 & \text{if } j \text{ is in the pack} \\ 0 & \text{otherwise} \end{cases} \text{ and } a_{ij} = \begin{cases} 1 & \text{if } i \in P_j \\ 0 & \text{otherwise} \end{cases}$$

It is assumed that  $h_{jk}$ 's are elements of the  $n \times n$  symmetric negative semi definite matrix  $H = (h_{jk})$ . In matrix form,  $(\mathbf{QPkP})$  can be written as

$$\begin{aligned} \text{Max } Z (= f(X)) &= DX + X^T H X \\ \text{subject to } AX &\leq b \end{aligned}$$

where  $X^T = (x_1, x_2, \dots, x_n)$  with  $x_j = 0$  or  $1, j = 1, 2, \dots, n$ . Here  $D = (d_1, d_2, \dots, d_n) \in R^n$  is a row vector,  $A$  is an  $m \times n$  matrix of zeros and ones and  $b^T = (1, 1, \dots, 1)$  is a row vector of ones.

### 2.1.1. Definitions

- **Pack Solution:** A solution  $X$  which satisfies (2.1) and (2.2) is said to be a pack solution.
- **Redundant Packing:** For any pack  $J$ , a column  $j^* \in J$  is said to be redundant if  $J - \{j^*\}$  is also a pack. If a pack contains one or more redundant columns, it is called a redundant pack. Column  $j^*$  is redundant with respect to the pack  $J$  iff  $\sum_{j \in J} a_{ij} \leq 2$  for all  $i \in P_{j^*}$ .
- **Prime Packing Solution:** A pack  $J^*$  is said to be a prime pack, if none of the columns corresponding to  $j \in J^*$  is redundant. A solution corresponding to the prime pack is called a prime packing solution.
- **Pseudo Concave Function:** Let  $f$  be a differentiable function defined on an open set  $T \subset R^n$ . Let  $S \subset T$  and  $X_1, X_2 \in S$ , then  $f$  is said to be pseudo concave if  $\nabla f(X_2)^T (X_1 - X_2) \leq 0 \Rightarrow f(X_1) \leq f(X_2)$ .

### 2.1.2 Following Theorems Forms the Basis for the Technique Developed

**Theorem 1:** [18] If  $J^* = \{j : x_j = 1\}$  is any prime pack of  $(\mathbf{QPkP})$  then  $X = \{x_j\}$  is an extreme point of the convex set formed by feasible region.

**Theorem 2:** If the objective function in  $(\mathbf{QPkP})$  has finite value then, there exists a prime packing solution where this value is attained.

**Proof:** Let a finite optimal solution of (QPkP) exists at  $X_0 \in S$  then the optimal value is

$$f(X_0) = DX_0 + X_0^T HX_0.$$

Let  $J_0$  be the pack corresponding to the solution  $X_0$ . If  $J_0$  is the prime, then it is done, otherwise a prime pack can be derived from  $J_0$  by dropping the redundant columns. Let  $J_1$  be the prime pack obtained from  $J_0$  and  $X_1$  be the corresponding solution of (QPkP) such that

$$f(X_1) = DX_1 + X_1^T HX_1.$$

Since  $J_1 \subseteq J_0$ , therefore,

$$DX_1 + X_1^T HX_1 \geq DX_0 + X_0^T HX_0.$$

or  $f(X_1) \geq f(X_0)$ .

As  $f(X_0)$  is the optimal value of  $f(X)$ , therefore,  $f(X_1) \leq f(X_0)$ . Hence  $f(X_1) = f(X_0)$ . Which proves that there exists a prime packing solution yielding the optimal value of the objective function of (QPkP).

**Theorem 3:** Let  $f(X)$  be a pseudo concave function defined on feasible set  $S$  and  $X^* \in S$  then  $X^*$  is an optimal solution for the program

$$\underset{X \in S}{\text{Maximize}} f(X)$$

if and only if,  $X^*$  is an optimal solution for the program

$$\underset{X \in S}{\text{Maximize}} \nabla f(X^*)^T X$$

where  $S$  is the feasible region.

**Proof:** Let  $X^*$  be an optimal solution for the program (QPkP), therefore,  $f(X^*) \geq f(X), \forall X \in S$ .

$$f(X) = f(X^* + X - X^*)$$

$$f(X^*) + \nabla f(X^*)^T (X - X^*) + \alpha(X^*, X - X^*) \square X - X^* \square$$

Where  $\alpha(X^*, X - X^*) \rightarrow 0$  as  $X \rightarrow X^*$

Since  $f(X^*) \geq f(X) \Rightarrow \nabla f(X^*)^T (X - X^*) + \alpha(X^*, X - X^*) \square X - X^* \square \leq 0$

And  $\alpha(X^*, X - X^*) \rightarrow 0$

$$\Rightarrow \nabla f(X^*)^T (X - X^*) \leq 0$$

$$\Rightarrow \nabla f(X^*)^T X \leq \nabla f(X^*)^T X^*$$

$\Rightarrow X^*$  is an optimal solution for the program  $Max \nabla f(X^*)^T X$ .

Conversely, let  $X^*$  be an optimal solution of  $Max \nabla f(X^*)^T X$  therefore,  
 $\nabla f(X^*)^T X^* \geq \nabla f(X^*)^T X, \forall X \in S$

$\Rightarrow \nabla f(X^*)(X - X^*) \leq 0, \forall X \in S$

Since  $f$  is a pseudo concave function, therefore,  $f(X) \leq f(X^*), \forall X \in S$ .

Hence  $X^*$  is an optimal solution for (QPkP).

**Theorem 4:** The objective function in (QPkP) is pseudo concave.

**Proof:** Given  $f(X) = DX + X^T HX, X \in S$ .

Since  $f$  is a differentiable function over an open set containing  $S$ , therefore, for any  $X_1, X_2 \in S = \{X \in R^n : AX \leq b, X \geq 0\}$ ,

$$\begin{aligned} (\nabla f(X_2)^T (X_1 - X_2)) &= (D + 2X_2^T H)(X_1 - X_2). \\ &= D(X_1 - X_2) + 2X_2^T H(X_1 - X_2). \\ &= (DX_1 - DX_2 + 2X_2^T HX_1 - 2X_2^T HX_2). \\ &= (DX_1 + X_1^T HX_1) - (DX_2 + X_2^T HX_2) + (2X_2^T HX_1 - X_2^T HX_2 - X_1^T HX_1) \\ &= f(X_1) - f(X_2) - (X_1 - X_2)^T H(X_1 - X_2). \\ &\Rightarrow \nabla f(X_2)^T (X_1 - X_2) + (X_1 - X_2)^T H(X_1 - X_2) = f(X_1) - f(X_2). \end{aligned}$$

Since  $H$  is a negative semi definite matrix, therefore,  $(X_1 - X_2)^T H(X_1 - X_2) \leq 0$ .

Hence,  $\nabla f(X_2)^T (X_1 - X_2) \leq 0 \Rightarrow f(X_1) - f(X_2) \leq 0 \Rightarrow f(X_1) \leq f(X_2)$ .

Thus,  $f$  is a pseudo concave function on  $S$ .

Following algorithm is developed to enumerate the given problem

#### Algorithm

**Step-1:** Given Quadratic Set Packing Problem (QPkP). Form the corresponding continuous program (QPkP') by embedding the feasible region into  $R^n$  (a cube with n vertices). Let  $S$  be the feasible set for (QPkP').

**Step-2:** Choose a feasible solution  $X_0 \in S$  such that  $\nabla f(X_0) \neq 0$ . Form the corresponding linear program (LP). On solving (LP) let  $X_1$  be its optimal solution. If  $X_1 = X_0$  and is of the 0-1 form, then this is the required solution of the given problem, otherwise let  $S_1 = \{X_1\}$ .

**Step-3:** Starting with the point  $X_1$ , form corresponding (LP), let its optimal solution be  $X_2 \neq X_1$ . Update  $S_1$  i.e.  $S_1 = \{X_1, X_2\}$ .

**Step-4:** Repeat step 3 for the point  $X_2$  and suppose at the  $i$ th stage  $S_i = \{X_1, X_2, \dots, X_i\}$ . Stop if at the  $(i+1)$ th stage  $X_{i+1} \in S_i$ , then  $X_{i+1}$  is the optimal solution of (QPkp').

**Step-5:** If  $X_{i+1}$  is an optimal solution of the form 0-1 then it is a solution of (QPkp) otherwise, go to **Step-6**.

**Step-6:** Apply Gomory cuts to find a solution of the 0-1 form and the corresponding prime pack solution.

**Note:** The algorithm must terminate after finite number of steps as it moves only on the vertices of the feasible region, which are finite in number, i.e. convergence is must.

### 2.3.2 Numerical Example

Consider the Quadratic Set packing problem (QPkp)

$$\begin{aligned} \text{Max } f(x) &= \begin{bmatrix} 30 & 20 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} -3 & 2 & 4 \\ 2 & -4 & 3 \\ 4 & 3 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= 30x_1 + 20x_2 + 10x_3 - 3x_1^2 - 4x_2^2 - 6x_3^2 + 4x_1x_2 + 8x_1x_3 + 6x_2x_3 \\ \text{subject to } & \quad x_1 + x_2 \leq 1 \\ & \quad x_2 + x_3 \leq 1 \\ & \quad x_1 + x_3 \leq 1 \\ & \quad x_1, x_2, x_3 = 0 \text{ or } 1 \end{aligned}$$

$$\text{where } J = \{1, 2, 3\}, \quad I = \{1, 2, 3\}$$

Here the objective function is pseudo concave therefore applying the algorithm developed above to this problem (QPkp).

The corresponding (QPkp') is

$$\begin{aligned} \text{Max } f(x) &= 30x_1 + 20x_2 + 10x_3 - 3x_1^2 - 4x_2^2 - 6x_3^2 + 4x_1x_2 + 8x_1x_3 + 6x_2x_3 \\ X = (x_1, x_2, x_3) \in S &= \{(x_1, x_2, x_3) \mid x_1 + x_2 \leq 1, x_2 + x_3 \leq 1, x_1 + x_3 \leq 1, x_1, x_2, x_3 \geq 0\}. \end{aligned}$$

**Step-2.** Choose  $X_0 = (1, 0, 0)$  as one of the feasible solution of (QPkp') with  $\nabla f(X_0) \neq 0$

The corresponding (LP) is

$$\text{Maximize } \nabla f(X_0)^T X = 24x_1 + 24x_2 + 18x_3 : X \in S.$$

It is a linear problem therefore can be solved by simplex method i.e.

$$\text{Max } f(x) = 24x_1 + 24x_2 + 18x_3$$

$$X = (x_1, x_2, x_3) \in S = \{(x_1, x_2, x_3) \mid x_1 + x_2 \leq 1, x_2 + x_3 \leq 1, x_1 + x_3 \leq 1, x_1, x_2, x_3 \geq 0\}.$$

Introduce the slack variables  $s_1, s_2, s_3$  in order to convert the constraints in to the equality

$$\text{Max } f(x) = 24x_1 + 24x_2 + 18x_3 + 0s_1 + 0s_2 + 0s_3$$

$$x_1 + x_2 + s_1 = 1$$

$$x_2 + x_3 + s_2 = 1$$

$$x_1 + x_3 + s_3 = 1$$

After applying the simplex algorithm the final optimal table is as follows:

**Table 1**

		$c_i$	24	24	18	0	0	0
$C_B$	B	$X_B$	$Y_1$	$Y_2$	$Y_3$	$s_1$	$s_2$	$s_3$
24	$x_1$	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
24	$x_2$	$\frac{1}{2}$	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
18	$x_3$	$\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
		33	0	0	0	15	9	9

Hence the optimal solution is  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ , which is not of the form 0 or 1, therefore apply the Gomory cut to get integer solution.

$$\text{The cut is } s_4 = \frac{-1}{2} + \frac{1}{2}x_4 + \frac{1}{2}x_5 + \frac{1}{2}x_6$$

Inserting this additional constraint in the optimal simplex table the next iterative table is

**Table 2**

			24	24	18	0	0	0	0
$C_B$	B	$X_B$	$Y_1$	$Y_2$	$Y_3$	$s_1$	$s_2$	$s_3$	$s_4$
24	$x_1$	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
24	$x_2$	$\frac{1}{2}$	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
18	$x_3$	$\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
0	$s_4$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1
		33	0	0	0	15	9	9	0

Now apply the dual simplex method, optimal iterative table is

**Table 3**

			9	8	9	0	0	0	0
$C_B$	B	$X_B$	$Y_1$	$Y_2$	$Y_3$	$s_1$	$s_2$	$s_3$	$s_4$
24	$x_1$	1	1	0	1	0	0	1	0
24	$x_2$	0	0	1	0	0	0	-1	1

Table 3

0	$s_2$	1	0	0	1	0	1	1	-1
0	$s_1$	0	0	0	-1	1	0	0	-1
		24	0	0	6	0	0	0	24

Hence the optimal solution of **(LP)** is  $X_1 = (1, 0, 0)$ , which is equal to  $X_0 = (1, 0, 0)$ , therefore this is the optimal solution for the original **(QPkP)** with optimal value 27.

**Note:** The function  $(-f)$  is said to be pseudo convex if  $f$  is pseudo concave, and  $\text{minimum}(-f) = \text{maximum } f$ .

Since the set partitioning problems have minimizing objective function therefore using above note, the algorithm so developed for set packing problem can also be applied to set partitioning problems.

## 2.2 Set Partitioning Problems

The Quadratic Set Partitioning Problem **(QPP)** is to find a partition of minimum cost subject to the condition that exactly one of the utility is satisfied. Mathematically, the quadratic **Set Partitioning Problem** is problem is-

$$(\text{QPP}) \text{ Min } Z (= f(X)) = \sum_{j=1}^n d_j x_j + \sum_{j=1}^n \sum_{k=1}^n h_{jk} x_j x_k$$

$$\text{Subject to } \sum_{j=1}^n a_{ij} x_j = 1, i \in I \quad (2.3)$$

$$x_j = 0 \text{ or } 1, j \in J \quad (2.4)$$

$$\text{Where } x_j = \begin{cases} 1 & \text{if } j \text{ is in the partition} \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad a_{ij} = \begin{cases} 1 & \text{if } i \in P_j \\ 0 & \text{otherwise} \end{cases}$$

It is assumed that  $d_j$  's are non-negative. A solution  $X$  which satisfy (2.3) and (2.4) is said a **partition solution**.

The algorithm developed to linearize Set Packing Problems is used to solve the Partition Problems since the objective function is pseudo convex. Following example is done accordingly.

### 2.2.1 Numerical Example

Consider the quadratic partitioning program **(QPP)**:

$$\begin{aligned}
 \text{Min } f(x) &= \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 \\ 2 & 4 & 3 \\ 4 & 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\
 &= 3x_1 + 2x_2 + x_3 + 3x_1^2 + 4x_2^2 + 6x_3^2 + 4x_1x_2 + 8x_1x_3 + 6x_2x_3 \\
 \text{subject to } & \quad x_1 + x_2 = 1 \\
 & \quad x_2 + x_3 = 1 \\
 & \quad x_1, x_2, x_3 = 0 \text{ or } 1
 \end{aligned}$$

Where  $J = \{1, 2, 3\}$  ,  $I = \{1, 2\}$

Here the objective function is pseudo convex therefore applying the algorithm developed as before to this problem **(QPP)**.

The corresponding **(QPP')** is

$$\begin{aligned}
 \text{Min } f(x) &= 3x_1 + 2x_2 + x_3 + 3x_1^2 + 4x_2^2 + 6x_3^2 + 4x_1x_2 + 8x_1x_3 + 6x_2x_3 \\
 X = (x_1, x_2, x_3) &\in S = \{(x_1, x_2, x_3) \mid x_1 + x_2 = 1, x_2 + x_3 = 1, x_1, x_2, x_3 \geq 0\}.
 \end{aligned}$$

**Step-2.** Choose  $X_0 = (0, 1, 0)$  as one of the feasible solution of **(QPP')** with  $\nabla f(X_0) \neq 0$

The corresponding **(LP)** is

$$\text{Minimize } \nabla f(X_0)^T X = 7x_1 + 10x_2 + 7x_3 : X \in S.$$

It is a linear problem therefore can be solved by simplex method i.e.

$$\begin{aligned}
 \text{Max } f(x) &= -7x_1 - 10x_2 - 7x_3 \\
 X = (x_1, x_2, x_3) &\in S = \{(x_1, x_2, x_3) \mid x_1 + x_2 = 1, x_2 + x_3 = 1, x_1, x_2, x_3 \geq 0\}.
 \end{aligned}$$

After applying the simplex algorithm the final optimal table is as follows:

**Table 4**

		$c_i$	-7	-10	-7
$C_B$	$B$	$X_B$	$Y_1$	$Y_2$	$Y_3$
-10	$x_2$	1	1	1	0
-7	$x_3$	0	-1	0	1
		-10	4	0	0

Hence the optimal solution of **(LP)** is  $X_1 = (0, 1, 0)$ , which is equal to  $X_0 = (0, 1, 0)$  therefore this is the optimal solution for the original **(QPP)** with optimal value 1.

### 3. CONCLUSIONS

Mathematical Programming techniques are of great interest and importance in various fields where an objective function is to be optimized under certain restrictions. There is no single method available for solving an optimization problem efficiently. Due to this reason a number of enumerative techniques have been developed.

Set Packing and Partitioning problems belong to a class of 0-1 integer programming problems. In this paper an extension of the earlier result [18] to set partitioning and set packing problems is presented.

The proposed linearization technique is justified analytically by mathematical results, the algorithms developed are supported by numerical examples.

## REFERENCES

1. Arora, S. R. and Puri, M. C. (1977): "Enumeration Technique for the Set Covering Problem with Linear Fractional Functional as its Objective Function", ZAMM, 56, 181-186.
2. Balas, E. (1980): "Cutting Planes from Conditional Bounds, A New Approach Set-Covering", Mathematical Programming Study, 12, 19-36.
3. Balas, E. and Padberg, M. W. (1976): "Set Partitioning: A Survey", SIAM Review, 18, 710-760.
4. Bazaraa, et. al. (1993): "Non Linear Programming: Theory and Algorithms", 2<sup>nd</sup> Edition John Wiley & Sons.
5. Bellmore, M. and Ratliff, H. D. (1971): "Set Covering and Involuntary Bases", Management Sciences, 18, 194-206.
6. Chvatal, V. (1979): "A Greedy Heuristic for the Set Covering Problem", Mathematics of Operations Research, 4, 233-235.
7. Feo, T. A. and Mauricio and Resende, G. C. (1989): "A Probabilistic Heuristic for Computationally Difficult Set Covering Problem", Operations Research Letters, 8, 67-71.
8. Fisher, M. L. and Wolsey, L. A. (1982): "On the Greedy Heuristic for Continuous Covering and Packing Problems", SIAM Journal on Algebraic and Discrete Methods, 3, 584-591.
9. Garfinkel, R.S. and Nemhauser, G.L. (1973): "Integer Programming, A Wiley Inter Science Publication", John Wiley and Sons.
10. Gupta, R. and Saxena, R. R (2014) "Set Packing Problem with Linear Fractional Objective Function", International Journal of Mathematics and Computer Applications Research, 4 (1), 9-18.
11. Hall, N. G. (1989): "A Fast Approximation Algorithm for the Multi Covering Problem", Discrete Applied Mathematics, 15(1), 35-40.
12. Huang, W. C.; Kao, C. Y. and Horng, J. T. (1994): "A Genetic Algorithm Approach for Set Covering Problem", IEEE International Conference on Genetic Algorithms: Proceedings, 569-574.
13. Lemke, C. E.; Salkin, H. M. and Spielberg, K. (1971): "Set Covering by Single Branch Enumeration with Linear Programming Sub Problem", Operations Research, 19 (4), 998-1022.
14. Nemhauser, G. L and Wolsey, L. A. (1999): "Integer and Combinatorial Optimization", Wiley Interscience Series in Discrete Mathematics and Optimization.
15. Padberg, M. W. (1974): "Perfect Zero-One matrices", Mathematical Programming, 6 (1), 180-196.

16. Padberg, M. (1993): "Lehman's Forbidden Minor Characterization of ideal 0-1 Matrices", Discrete Mathematics, 111 (1-3), 409-410.
17. Roth, R. (1969): "Computer Solution to Minimum Cover Problem", Operations Research, 17, 455-465
18. Saxena, R. R. and Arora, S. R. (1995): "Enumeration Technique for Solving Multi Objective Linear Set Covering Problem", APZOR, 12, 87-97.
19. Saxena, R. R. and Arora, S. R. (1996): "A Linearization Technique for Solving the Quadratic Set Covering Problem", Optimization, 39, 1, 35-42.
20. Saxena, R. R. and Arora, S.R. (1998): "Cutting Plane Technique for the Multi-Objective Set Covering Problem with Linear Fractional Objective Functions", IJOMAS, 14, 1, 111-122.
21. Saxena, R. R and Gupta, R. (2012) "Linearization Technique for Solving Quadratic Fractional Set Covering Partitioning and Packing Problems", International Journal of Research in Engineering and Social Science, 2, (2), 49-87.

